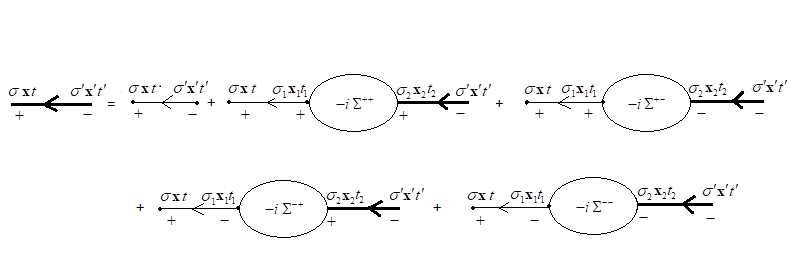
**Self-Energy**

**Self Energy for GFR (possibly time-dependent potential)**

We’ll can examine the self energy concept in the context of the general contour ordered GF, which may include time-dependent interactions. The self energy contains the energy corrections, lifetime corrections, and spectral weight of the excitations in question. It is the same as the information obtained from doing the TDPT in QM1. So consider the self-energy expansion for G+-, for instance,



Basically, Σab begins with an ‘a’ vertex, and ends with a ‘b’ vertex, where ‘a’ and ‘b’ = ±. But we can have any kind of vertex in between. Some of the self-energies would look like, for example,



We could include external field interactions as well, if desired. Note by their nature, they could only come in with Σ++ and Σ-- (could we though? ‘cause wouldn’t they be cutable?). Translating our diagrammatic equation for G+-  into math, we have:



And similarly for the other 3 GF. Clearly, we cannot so far write any *1* of the GF in terms of itself and a self-energy. This is the advantage of working with causal GF when we can. And a disadvantage of working with the NEGF. But if we were to write out all 4 of these equations, we would find that we could combine them in matrix form,



with



Additionally, we would have that the free GF’s satisfy the differential equation (see Formal Properties file),



and if we apply this fact to the equation above, then we’d have,



Despite the symmetry of the formulation, it doesn’t take into account the fact that the GF’s are linearly related. To reduce the equations to the only degrees of freedom, we can make the following matrix transformation on the equations. We define:



Then the transformed matrix, G´, is:



Now from the relations in the Formal properties file:



this simplifies to:



where we define F = G-- + G++. The transformed self energies are, Σ´:



To make progress on this, we can use the following relationship:



which will eliminate the lower right matrix element, as can see. Before proceeding, let’s prove that relationship. So we’ll start from which follows from: GC + GAC = G> + G<. Then if we insert the self energy equations GC,AC,>,< into this relationship (leaving summations over indices and position/time integrals implicit) we have:



and then apply the differential operator (i∂t – H0) to each side of the equation. And we’ll get, using some notational license:



But now recall from formal properties that:



which implies,



and so the last line simplifies to:



and so we have,



which is what we wanted to prove. Okay now let’s go back to the self-energy equation and use this relationship. We’ll have:



where we define Ω = Σ-- + Σ++, ΣR = Σ-- + Σ-+, and ΣA = Σ-- + Σ+-. The usefulness of this formulation lies in the fact that some of the equation decouple. Consider the recursive matrix GF equation again, leaving the summations, integrals implicit.



Inserting our R’s, we have:



And inserting our results for these guys in, leaving integrals implicit, again,



The nice thing about this is that the equations for GA and GR are entirely self-consistent, excepting the fact that their respective self-energies involve other GF’s anyway. So explicitly we have:



Therefore, we *can* write out a self-consistent equation for GA, or GR in terms of just itself. If we operate on both sides of the R equation with the free particle Schrodinger operator we get,



which gives us an integro-differential equation from which one might attempt a non-perturbative calculation of G. You might think of the integral term as a sort of self consistent single particle potential approximation to the interaction. One could attempt to solve this equation iteratively. An advantage of this approach is that we have an integral equation for GR that is entirely separated out, unlike the more formal coupled equation we had in the Formal Properties file. The disadvantage is that this equation is not exact unless we include *all* self energy terms in Σ. And if we just include, say, the first order term in Σ, then our equation is only good to first order. A non-perturbative approximation is often done by making a self-consistent approximation on Σ in terms of G itself. Of course that makes the equation harder to solve.

For time-independent homogeneous systems (this would now exclude the possibility of single particle potentials within Σ) G and Σ will be functions purely of the difference of their position and time arguments, and so we can go to Fourier space and obtain (see Fourier transfrom file maybe),



Matrix inversion of this equation yields,



where 1/ and -1 would mean matrix inversion in this context. And if we can presume G0 is isotropic in spin space, and diagonal in the momentum basis, then we can simplify to this expression.



and it would follow likewise that:



Given this form, we can obtain the spectral function:



The poles of the Green’s function determine the excitations, and so we will want to expand the Green’s function about its pole.



Suppose we find the root of this equation to be . Then we’ll expand the denominator of the Green’s function,



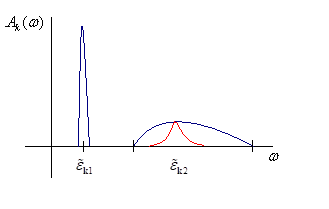
which we can write as:



Z(k) is called the renormalization factor. Note that from the formal properties file, we have ∫dωAkk(ω) = [ck, ck†] = 1. And so this would indicate that the closer the residue Zk is to 1 (it’s always less than 1), the closer our pole is to being an exact excitation. And we would identify as the energy correction, and as -Γ/2, half the decay rate. We can also examine the spectral function, which will give these identifications another light. The spectral function will come to:



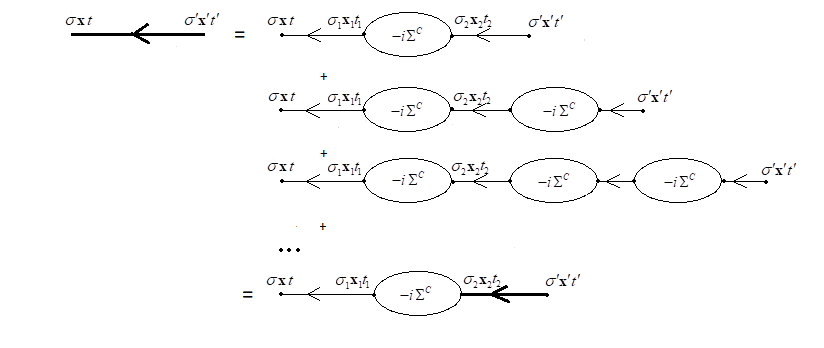
and again, the identifications make sense. Note how ImΣR must be less than 0 for A to be positive. A typical spectral function, A(ω) will look like below, in blue



The two peaks indicates that there are two excitations of the system associated with momentum, k. The width of the peak is given by the ImΣ. So the delta function thing on the left would have that ImΣ = 0, while not so for the right hand curve. If we tried to approximate the latter with a Lorentzian, as we do above, we’d have the red curve, whose location gives the energy best associated with it, and whose width would give the scattering rate basically, and whose height would roughly give the renormalization factor, basically the amount that this would resemble a quasi-particle.

**Self-Energy for GC (time-independent interaction, |GS> expectation)**

The same sort of analysis holds for the causal GF in the time-independent perturbation case, as it can be solved for all by itself [again we’ll presume spin-independent interactions].



In real space, we’d have quite generally:



and applying our free GF operator on each side, we’d get:



Presuming homogeneity in space and time (so H0 is free particle Hamiltonian), G will be a function only of the difference of its spatial and temporal arguments. So then we can take the Fourier transform of both sides (see Fourier transform file perhaps),



and solve for G. We get:



where -1 means matrix inverse. Or more succinctly, but less explicitly,



where 1/ really means matrix inverse and those first three terms in the numerator are really proportional to the unit matrix δσσ´. And we’ll recall δk = sgn(εk-εF). This addition is necessary to get the correct time-dependence – see non-interacting GF’s. Turns out that ΣC will also be proportional to sgn(εk-εF). Otherwise, the analytic properties of GC I’d expect to be similar to GAR above. Finally, if we can presume isotropy in spin space, then everything will be independent of spin, and we can say,

